s-Wave K-N Scattering by the N/D Method

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The K-N s-wave scattering lengths and effective-range parameters have been calculated for both isotopic spin states, using the N/D method. The nearby part of the interaction cut has been approximated by two poles, whereas following Balázs' procedure the entire distant cut has been replaced by two effective-range poles at predetermined positions, but with unknown residues. The residues of the effective-range poles are determined by matching the N/D amplitude with that calculated with a fixed energy dispersion relation, at two properly chosen points. In calculating the latter amplitudes only the contributions of Λ, Σ, Y_1^*, ρ, and ω exchange terms are retained and the values of Y_1^*, ρ, and ω coupling constants are taken from a unitary symmetry scheme. The calculated values of scattering length, effective range, and low-energy phase shifts for the I=1 state are in good agreement with the experimental results of Goldhaber *et al.* For the I=0 state, the calculated phase shifts are consistent with the large s-wave solution of Goldhaber *et al.*

1. INTRODUCTION

FOLLOWING Balázs' work,[1] a number of investigators[2–4] have successfully applied the effective-range two-pole approximation for the distant cross-channel singularities in the N/D analysis of meson-meson and meson-nucleon scattering. The immediate advantage of this approximation scheme is that it does not introduce any arbitrary parameters into the theory. The present paper is devoted to the study of kaon-nucleon s-wave amplitude using the aforementioned technique. We choose to work in the s plane following Singh and Udgaonkar.[2]

In Sec. 2, we mention the singularity structure of the partial wave amplitude in the s plane. In Sec. 3, the N/D amplitude has been set up. The short Λ, Σ cuts have been replaced by two poles whose positions and residues are obtained from the discontinuities across these cuts. Moreover, all other interaction singularities have been approximated by the two effective-range poles and their positions have been chosen *a priori.* Thus, the N function effectively contains only four simple poles. The residues at the two effective-range poles are determined by matching the N/D amplitude with that calculated with a fixed-energy dispersion relation. The latter amplitude is given in Sec. 4. The N/D amplitude being thus determined, we have calculated in Sec. 5 the s-wave scattering length, the effective range, and the low-energy phase shifts and compared them with experimental data.

2. SINGULARITY STRUCTURE

In addition to the unitarity cut extending over the entire physical region, the K-N partial-wave amplitude has the following singularity structure in the s plane: (1) Short cuts extending from $[(M_N^2-M_K^2)/M_{\Lambda,\Sigma,Y^*}]^2$ to $2(M_N^2+M_K^2)-M_{\Lambda,\Sigma,Y^*}{}^2$ corresponding to the exchange of Λ, Σ, and Λ-π resonances in u channel. (2) Short cuts extending from $(M_N^2-M_K^2)$ to $(M_N^2+M_K^2-\frac{1}{2}M_{\rho,\omega,\phi}{}^2)+\frac{1}{4}[(4M_N^2-M_{\rho,\omega,\phi}{}^2)(4M_K^2-M_{\rho,\omega,\phi}{}^2)]^{1/2}$ and a circular cut of radius $(M_N^2-M_K^2)$ around the origin, arising from the exchange of various pion resonances in t channel. (3) The distant cut extending over the entire negative s axis, to which all the exchange processes mentioned above contribute. (4) A cut along the entire negative s axis and possibly a pole at the origin arising from the kinematics.

[1] L. A. P. Balázs, Phys. Rev. **128**, 1939 (1962).
[2] V. Singh and B. M. Udgaonkar, Phys. Rev. **130**, 1177 (1963).
[3] V. Singh and B. M. Udgaonkar, Phys. Rev. **128**, 1820 (1962).
[4] S. K. Bose and S. N. Biswas, Phys. Rev. **134**, B635 (1964).

3. DETERMINATION OF THE N/D AMPLITUDE

We use the standard normalization for the s-wave amplitude,[5] i.e.,

$$f_{0+}=\exp i\delta_{0+}\sin\delta_{0+}/q\,, \tag{1}$$

and write it in the N/D form

$$f_{0+}{}^{1,0}(s)=N^{1,0}(s)/D^{1,0}(s)\,, \tag{2}$$

where $D(s)$ contains the unitarity cut and $N(s)$ contains all the interaction singularities. The superscripts 1 and 0 refer to the two isotopic spin states. In calculating the effect of the short interaction cuts only the contribution from Λ and Σ exchange has been considered,[6] whereas the contribution of all other interaction cuts as well as the kinematic singularities are approximated by the two effective-range poles.

TABLE I. Positions and residues of the two nearby poles.

The total isotopic spin I	Pole position	Residues
1	18	0.42
	45	−0.29
0	18	0.83
	45	−0.50

[5] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960). For the relation of $f_{0+}(s)$ with the invariant A and B amplitudes, see Eqs. (2.21), (2.22), and (2.23) of Ref. 5.
[6] The contribution of the short Λ, Σ cut turns out to be much smaller than that of the effective-range poles so that our result will not differ appreciably from what a two-pole effective-range approximation alone would predict.

The contribution of the Σ-exchange term to the s-wave amplitude is given by

$$f_{0+}{}^{\Sigma:1,0}(s)=\binom{1}{3}\frac{g_{K\Sigma N}{}^2}{8\pi}\left\{\frac{[(s^{1/2}+M_N)^2-M_K{}^2][s^{1/2}+M_\Sigma-2M_N]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}Q_0\left(1+\frac{2s[2(M_N{}^2+M_K{}^2)-M_\Sigma{}^2-s]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}\right)\right.$$
$$\left.+\frac{[(s^{1/2}-M_N)^2-M_K{}^2][s^{1/2}+2M_N-M_\Sigma]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}Q_1\left(1+\frac{2s[2(M_N{}^2+M_K{}^2)-M_\Sigma{}^2-s]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}\right)\right\}, \quad (3)$$

where $g_{K\Sigma N}$ is the rationalized pseudoscalar coupling constant, $Q_l(x)$ are the Legendre functions of the second kind and the multiplicative factors 1 and 3 correspond to the isospin 1 and 0 states of the s-wave amplitude. $f_{0+}{}^{\Lambda:1,0}(s)$ is obtained from (3) on replacing $g_{K\Sigma N}$ and M_Σ by $g_{K\Lambda N}$ and M_Λ, respectively, and setting both the isospin factors equal to unity. This gives for the short Σ cut,

$$\mathrm{Abs}f_{0+}{}^{\Sigma:1,0}(s)=\binom{1}{3}\frac{(-g_{K\Sigma N}{}^2)}{16}\left\{\frac{[(s^{1/2}+M_N)^2-M_K{}^2][s^{1/2}+M_\Sigma-2M_N]}{[s-(M_N-M_K)^2][s-(M_N+M_K)^2]}\right.$$
$$\left.+\frac{[(s^{1/2}-M_N)^2-M_K{}^2][s^{1/2}+2M_N-M_\Sigma]}{[s-(M_N-M_K)^2][s-(M_N+M_K)^2]}\left(1+\frac{2s[2(M_N{}^2+M_K{}^2)-M_\Sigma{}^2-s]}{[s-(M_N-M_K)^2][s-(M_N+M_K)^2]}\right)\right\}, \quad (4)$$

and a similar expression for $\mathrm{Abs}f_{0+}{}^{\Lambda:1,0}(s)$.

The short Λ and Σ cuts are almost overlapping. The length of this cut is about half its average distance from threshold. Moreover, the absorptive parts have opposite signs over two more or less equal regions of the cut, so that a single-pole approximation would not be adequate. Hence, a two-pole approximation has been made for the short Λ, Σ cut to account for the contributions from the two regions of the cut, where the absorptive parts have opposite signs. Following an analysis of K^+-photoproduction data by Moravscik[7] we have set both $g_{K\Sigma N}{}^2/4\pi$ and $g_{K\Lambda N}{}^2/4\pi$ equal to unity. It is realized that unitary symmetry when applied to pseudoscalar meson-baryon coupling strengths would suggest values for the K-Λ and K-Σ coupling constants of the same order as the π-N coupling constant. But, as suggested by Sakurai,[8] it is still possible to get to the experimental values of $g_{K\Sigma N}$ and $g_{K\Lambda N}$ within the framework of unitary symmetry, if the latter is applied to the pseudovector coupling constants and the pseudoscalar coupling constants are treated as phenomenological parameters.

Then following Frautschi and Walecka[5] the effective pole positions are fixed near the maxima of the absorptive parts in the two regions mentioned above and the residues are evaluated as the integrals of the absorptive parts over corresponding regions. The values are given[9] in Table I.

The approximate positions of the two effective-range poles are determined by the procedure suggested by Balázs.[1] The values are -20 and -500. Thus, the s-wave amplitude $f_{0+}{}^{1,0}(s)$ can now be expressed as a sum of four pole terms and an integral over the unitarity cut. Then following (2), the $N(s)$ function is expressed as a sum of the effective pole terms and $D(s)$ as an integral over the unitarity cut. This gives

$$N^{1,0}(s)=\binom{0.42}{0.83}\frac{1}{s-18}-\binom{0.29}{0.50}\frac{D(45)}{s-45}+\binom{R_1{}^1}{R_1{}^0}\frac{1}{s+500}+\binom{R_2{}^1}{R_2{}^0}\frac{1}{s+20}, \quad (5)$$

where the $D(s)$ function has been normalized to unity at $s=18$. The subtracted dispersion relation for $D(s)$ reads

$$D^{1,0}(s)=1-\frac{s-18}{\pi}\int_{(M_N+M_K)^2}^{\infty}ds'\frac{q(s')N^{1,0}(s')}{(s'-s)(s'-18)}$$
$$=1-\frac{s-18}{\pi}\int_{(M_N+M_K)^2}^{\infty}ds'\frac{[s'-(M_N+M_K)^2]^{1/2}[s'-(M_N-M_K)^2]^{1/2}N^{1,0}(s')}{(s')^{1/2}(s'-s)(s'-18)}. \quad (6)$$

The above integral equation can be solved[10] for $s=45$, and substituted into (5) so that the $N(s)$ and $D(s)$ functions

[7] M. Moravscik, Phys. Rev. Letters **2**, 352 (1959).
[8] J. J. Sakurai, Proceedings of the International School of Physics, Varenna, Italy, 1963 (unpublished).
[9] All the numerical values are in pion-mass units unless mentioned otherwise.
[10] For evaluating the right-hand-side integral of Eq. (6), we haveapproximated q by $\frac{1}{2}[1-(M_N-M_K)^2/2s'][s'-(M_N+M_K)^2]^{1/2}$. Since the minimum value of s' in the region of integration is $(M_N+M_K)^2$, the error introduced by neglecting the higher order terms is less than 1%.

are obtained in terms of the only two unknown residues R_1 and R_2. These residues are now determined by matching the N/D amplitude against the amplitude obtained from fixed-energy dispersion relation at two points on the singularity-free region of the s axis. The matching points are chosen at $s=90$ and $s=104$.

4. FIXED-ENERGY DISPERSION RELATION

The fixed-energy dispersion relation satisfied by the invariant amplitude $A(s,t,u)$ is

$$A(s,t,u)=\frac{R_\Lambda}{M_\Lambda^2-u}+\frac{R_\Sigma}{M_\Sigma^2-u}+\int_{(M_\Lambda+1)^2}^{\infty}\frac{A_u(u')}{u'-u}du'+\int_4^\infty\frac{A_t(t')dt'}{t'-t}. \tag{7}$$

A similar relation holds for $B(s,t,u)$. Using (7), we calculate $A(s,t,u)$ by assuming that the u' and t' integrals are exhausted by the contributions from their low-lying resonating states,[11] i.e., $Y_1^*(1385)$, ρ, and ω. $B(s,t,u)$ is similarly calculated. With A and B so obtained, we can easily calculate the s-wave amplitude $f_{0+}(s)$. This has the form

$$f_{0+}{}^{1,0}(s)=f_{0+}{}^{\Lambda:1,0}(s)+f_{0+}{}^{\Sigma:1,0}(s)+f_{0+}{}^{Y_1^*:1,0}(s)+f_{0+}{}^{\rho:1,0}(s)+f_{0+}{}^{\omega:1,0}(s). \tag{8}$$

The Λ, Σ contributions are given in (3). The Y_1^* contribution is given by

$$f_{0+}{}^{Y_1^*:1,0}(s)=\binom{1}{3}\frac{g_{KY_1^*N}{}^2}{8\pi}\left\{\frac{[(s^{1/2}+M_N)^2-M_K{}^2][X+(s^{1/2}-M_N)Y]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}Q_0\left(1+\frac{2s[2(M_N{}^2+M_K{}^2)-M_{Y_1^*}{}^2-s]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}\right)\right.$$
$$\left.+\frac{[(s^{1/2}-M_N)^2-M_K{}^2][-X+(s^{1/2}+M_N)Y]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}Q_1\left(1+\frac{2s[2(M_N{}^2+M_K{}^2)-M_{Y_1^*}{}^2-s]}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}\right)\right\}, \tag{9}$$

where

$$X=(M_{Y_1^*}+M_N)\left\{M_N{}^2-\frac{(s+M_{Y_1^*}{}^2)}{2}+\frac{M_{Y_1^*}{}^2-M_N{}^2}{3}+\frac{1}{6M_{Y_1^*}{}^2}(M_{Y_1^*}{}^2-M_N{}^2+M_K{}^2)^2\right\}-\frac{(M_{Y_1^*}{}^2-M_N{}^2)^2-M_K{}^4}{6M_{Y_1^*}}, \tag{9a}$$

$$Y=\left\{\frac{s+M_{Y_1^*}{}^2}{2}-M_N{}^2-\frac{M_K{}^2}{3}+\frac{2M_N(M_{Y_1^*}+M_N)}{3}-\frac{M_N}{3M_{Y_1^*}}(M_{Y_1^*}{}^2-M_N{}^2+M_K{}^2)\right.$$
$$\left.-\frac{1}{6M_{Y_1^*}{}^2}(M_{Y_1^*}{}^2-M_N{}^2+M_K{}^2)^2\right\}. \tag{9b}$$

The ρ contribution is given by

$$f_{0+}{}^{\rho:1,0}(s)=\binom{1}{-3}\frac{1}{8\pi}\left\{\frac{[(s^{1/2}+M_N)^2-M_K{}^2][X'+(s^{1/2}-M_N)Y']}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}Q_0\left(1+\frac{2sM_\rho{}^2}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}\right)\right.$$
$$\left.+\frac{[(s^{1/2}-M_N)^2-M_K{}^2][-X'+(s^{1/2}+M_N)Y']}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}Q_1\left(1+\frac{2sM_\rho{}^2}{[s-(M_N+M_K)^2][s-(M_N-M_K)^2]}\right)\right\}, \tag{10}$$

where

$$X'=-g_{KK\rho}g_{NN\rho}'(2M_N{}^2+2M_K{}^2-2s-M_\rho{}^2)/2M_N, \tag{10a}$$

$$Y'=-2(g_{KK\rho}g_{NN\rho}+g_{KK\rho}g_{NN\rho}'), \tag{10b}$$

where g and g' refer to rationalized coupling strengths for charge and magnetic moment couplings, respectively. The ω contribution is obtained by replacing the subscript ρ by ω throughout (10) and setting both the isospin factors equal to unity.

The values of the ρ, ω, and Y_1^* coupling constants have been calculated from the known data on ρ, $N^*(1238)$ decay widths and the electromagnetic form factors of the nucleons, by applying a unitary symmetry scheme.[12] Here ρ, ω, and K^* are taken as the members of a vector meson octet and Y_1^*, together with N^*, Ξ^*, and the newly discovered Ω^-, forms a Baryon decuplet. The calculated values of the coupling strengths are

$$\begin{aligned} g_{Y_1^*KN}{}^2/4\pi&=(0.05),\\ g_{KK\rho}g_{NN\rho}/4\pi&=(0.56),\\ g_{KK\rho}g_{NN\rho}'/4\pi&=(2.25),\\ g_{KK\omega}g_{NN\omega}/4\pi&=(1.68),\\ g_{KK\omega}g_{NN\omega}'/4\pi&=0. \end{aligned} \tag{11}$$

[11] Following the strip approximation, Singh and Udgaonkar (Ref. 2) have shown that the high-energy contributions from the two crossed channels are equivalent to a contribution from low energies in the direct channel. The latter again may be approximated by the contributions from low-energy resonances. In the absence of any such resonances in the direct channel, we expect the contributions of high energies in the crossed channels to be small, and so neglect them. See also L. A. P. Balázs, Phys. Rev. **134**, B1315 (1964).

[12] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964).

5. RESULTS AND DISCUSSION

The two parameters R_1 and R_2 being thus determined, $N(s)$ and $D(s)$ are now known functions of energy. Then the scattering length a and effective-range parameter r are obtained from

$$\frac{1}{a}=(q\cot\delta)_{\text{threshold}}=\left(\frac{\mathrm{Re}D(s)}{N(s)}\right)_{s=(M_N+M_K)^2} \tag{12}$$

and

$$\tfrac{1}{2}r=\left[\frac{\partial}{\partial q^2}(q\cot\delta)\right]_{\text{threshold}}$$
$$=\left[\frac{\partial}{\partial q^2}\left(\frac{\mathrm{Re}D(s)}{N(s)}\right)\right]_{s=(M_N+M_K)^2}. \tag{13}$$

The calculated values for $\mathrm{I}=1$ are

$$a^1=-0.34\text{ F},\quad r^1=0.25\text{ F}.$$

An s-wave effective-range fit to the experimental K^+-p scattering data by Goldhaber *et al.*[13] gives $a^1=-0.29\pm0.015$ F, and $r^1=0.5\pm0.15$ F. The s-wave phase shift has also been calculated as a function of the laboratory momentum and compared against the experimental value of the above authors (Fig. 1). The agreement is quite good.

For $\mathrm{I}=0$, the calculated values for the scattering length and effective range are $a^0=-0.38$ F, $r^0=0.10$ F. Goldhaber *et al.*[14] have analyzed the K^+-d scattering data in a limited energy range, and obtained two sets of solutions for the s-wave phase shifts for $\mathrm{I}=0$. Our calculated values support the large s-wave solution (set B in Table II of Ref. 14). This solution is consistent with a scattering length of about -0.4 F. On the other hand, their set A solution and a similar solution obtained earlier by Rodberg and Thaler[15] from K^+ interaction in emulsion suggest values for scattering length and phase shifts much smaller[16] than ours. However, it must be mentioned that the experimental data for this case are very meager and admit extremely large uncertainties, so that no reliable comparison can be made at present.

The calculated values of the scattering lengths for the two isospin states are seen to be about the same, because the Y_1^* and Σ contributions to the s-wave amplitude partly cancel each other, and ρ contribution is small, so that the predominant contributions come from Λ and ω exchange which contribute equally to both the isospin states. To check the sensitivity of the result on the choice of matching points, we have alternatively determined the residues by matching the amplitude and its derivative at $s=90$ and $s=104$ separately. The values of the scattering length obtained for these two cases agree with the original value within 20%.

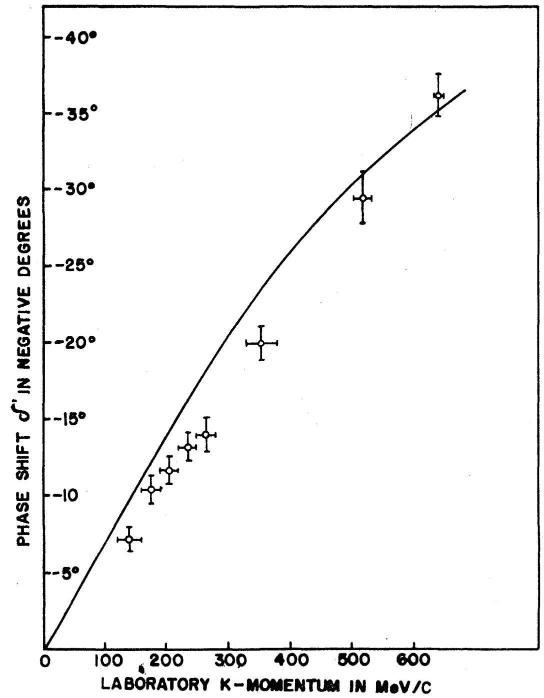

FIG. 1. Plot of s-wave K-N phase shift for $\mathrm{I}=1$ against laboratory K momentum. The open circles represent experimental points.

In the present investigation, as in the earlier work of Costa *et al.*[17] one comes across a spurious zero in the $D(s)$ function, in the unphysical region. However, if we determine the residue at this pole, it turns out to be negative so that it does not correspond to any bound state in K-N system. This is similar to the effect that one observes in potential theory where for strong repulsion, pole approximations to the left cut always lead to the appearances of ghost states in the theory.[18]

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[13] S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, T. F. Stubbs, G. M. Pierrou, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **9**, 135 (1962).

[14] W. Slater, D. H. Stork, H. K. Ticho, W. Lee, W. Chinowsky, G. Goldhaber, S. Goldhaber, and T. O'Halloran, Phys. Rev. Letters **7**, 378 (1961).

[15] L. S. Rodberg and R. M. Thaler, Phys. Rev. Letters **4**, 372 (1960).

[16] M. M. Islam, Nuovo Cimento **20**, 546 (1961), has obtained a fit to the small s-wave phase-shift data by restricting the t-channel contribution to the ρ pole alone.

[17] G. Costa, R. L. Gluckstern, and A. H. Zimmerman, in *Proceedings of the 1962 International Conference in High Energy Physics, CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 361.

[18] S. C. Frautschi, *Regge Poles and S-Matrix Theory* (W. A. Benjamin, Inc., New York, 1963), pp. 15–16.